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# VALUE ADDED LINEAR OPTIMIZATION OF RESOURCES (VALOR)

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PREPARED BY  
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**VALUE ADDED LINEAR OPTIMIZATION OF RESOURCES  
(VALOR)**

March 1992

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**VALUE ADDED LINEAR  
OPTIMIZATION OF RESOURCES  
(VALOR)**

**SUMMARY  
CAA-TP-92-1**

**THE REASON FOR PERFORMING THIS RESEARCH** was to formulate a math programming algorithm that could be used to perform acquisition strategy optimization that dynamically incorporates nonlinear "learning curve" costs for use in Value Added Analysis. An approximate mixed integer programming (MIP) formulation was devised for this purpose.

**THE SPONSOR** was the Director, US Army Concepts Analysis Agency.

**THE OBJECTIVES** were to:

- (1) Identify the need for dynamic learning curve costing in acquisition strategy optimization.
- (2) Formulate a specific MIP for computer solution.
- (3) Implement performance-improving measures to speed the solution of the model.
- (4) Report on the performance and characteristics of the model implementation.

**THE SCOPE OF THE PAPER** was limited to analysis of the major item systems under consideration for procurement in the Army Program Value Added Analysis 94-99 (VAA Phase II) Study.

**THE MAIN ASSUMPTION** of this work is: learning curve costs can be described as an exponential function of the cumulative number of items produced.

**THE BASIC APPROACH** used in this analysis was to formulate a MIP with the objective of maximizing the effectiveness of the force subject to constraints on budget, force structure, and production capabilities. Additional constraints were added to improve computational performance.

**THE PRINCIPAL FINDINGS** of the work reported herein are:

(1) Approximate nonlinear learning curve costs can be calculated in a mixed integer programming algorithm.

(2) The performance of the mixed integer programming model used for cross mission area acquisition strategy is such that extremely fast response can be given to "what-if" type questions from study sponsors.

**THIS EFFORT** was directed by LTC Andrew G. Loerch, Force Systems Directorate.

**COMMENTS AND QUESTIONS** may be sent to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-FSR, 8120 Woodmont Avenue, Bethesda, MD 20814-2797.

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## VALUE ADDED LINEAR OPTIMIZATION OF RESOURCES

### CHAPTER 1

#### INTRODUCTION AND BACKGROUND

**1-1. INTRODUCTION.** Each year, the United States Army procures billions of dollars worth of weapons and equipment so that it can accomplish its worldwide mission of deterrence. The process of deciding what to buy, how much, and when to best respond to the threat is extremely complex, requiring extensive analysis. Recent work in the area of Army procurement has led to a return on investment (ROI) approach to acquisition decisions. The ROI approach uses cost-benefit analysis as a means of determining relative return. Two common techniques used in this analysis are mathematical programming and cost estimation. Although they are related through constraints on available procurement funds, the use of nonlinear cost learning curves, which accurately represent system costs as a function of quantity produced, have not been incorporated into the mathematical programming formulations that compute the quantities of items to be procured. As a result, the solutions obtained could be either suboptimal, or even infeasible with respect to budgetary limitations. In this paper, we present the Value Added Linear Optimization of Resources (VALOR) Model, a mixed integer linear programming formulation that uses a piecewise linear approximation of the learning curve costs for a more nearly accurate portrayal of budgetary constraints.

#### 1-2. LEARNING CURVES

a. Learning curves are used to mathematically represent the concept that the more items of a particular type a factory produces, the less each item will cost. Also known as "progress curves," "improvement curves," and "experience curves," they were developed for use in the aircraft industry before and during World War II. Since then, the technique has spread to many other industries. There have been many publications describing applications, justifications, and forms for learning curves. In 1936, Wright described the use of learning curves in the aircraft industry.<sup>16</sup> Since then, literally hundreds of articles on learning curves have been published. Dutton, et al., review about 300 such articles.<sup>5</sup>

b. The current atmosphere of defense cuts requires that closer attention be paid to weapon systems costing. The front page of the Washington Post on November 9, 1991,<sup>13</sup> carried the following headline:

"Cuts in Defense Budget Create New Inefficiencies."

The article accompanying this headline describes how reducing the total quantities procured of particular weapons increases the unit costs. This effect results in less savings than anticipated from defense cuts. Learning curve

effects that influence the costs of weapon system acquisition alternatives must be considered to better understand the impact of particular changes in defense investment.

c. The Federal government in general, and the Department of Defense in particular has mandated the use of learning curve costing for cost estimation of acquisition systems.<sup>4</sup> Each government contractor must submit a DD Form 1921-2 quantitatively describing the anticipated learning behavior of their manufacturing process. These data are used by cost analysts throughout the acquisition process for determining contract prices, budgetary projections, and cost and effectiveness analyses.

d. Many forms exist to mathematically depict learning effects. Probably the most popular is the so-called power, or exponential, form which is represented as follows:

$$C(y) = Ay^{-b}, \quad (1)$$

where

y = the cumulative number of items produced,  
 C(y) = unit cost of the y<sup>th</sup> item produced,  
 A = the cost of the first unit produced,  
 b = the learning parameter.

e. Kanton and Zangwill<sup>11</sup> have suggested that this form of the learning curve is deficient in that it cannot remain form-invariant under aggregation of costs over the subcomponents of the item. However, Stump<sup>15</sup> describes and justifies a method to estimate composite learning curves in the power form, overcoming this objection. This estimate seems to do well for computing costs at the system, rather than the component, level, and the system view is regarded as the appropriate one for the Department of the Army program and budget development.

**1-3. OPTIMIZATION OF ACQUISITION STRATEGY.** Mathematical programming is frequently used to determine an optimal, in some sense, funding and acquisition stream for procurement of Army equipment. In this paragraph, several of these efforts are described.

a. In 1984, the Resource Constrained Procurement Objectives for Munitions (RECPOM-85) Study was performed by the US Army Concepts Analysis Agency (CAA).<sup>8</sup> Its purpose was to develop an optimization model that could be used to calculate the best mix of ammunition to procure such that the effectiveness of the force would be maximized. Prior to this effort, requirements for ammunition were computed in an unconstrained manner. These unconstrained requirements were then modified to take into account the fact that sufficient funds were not available to buy the entire requirement. The modifications that were made lacked strong analytical underpinnings and RECPOM was performed to overcome this deficiency. A stated limitation of the

RECPOM optimization model was that it could not consider the unit costs of the ammunition as a function of number of items produced. As an alternative, an average unit cost of the items was used that did not vary with quantity. This limitation was never resolved and greatly limited the usefulness of the methodology.

b. A more general methodology for optimizing acquisition strategy was developed as part of the Army Aviation Modernization Tradeoff Requirements (AAMTOR) Study, a joint effort between CAA and the Naval Postgraduate School.<sup>2,6</sup> The optimization model, known as Phoenix, is a large-scale, mixed integer program whose objective is to find the minimum cost set of equipment quantities, as well as finding the best, with respect to cost, timing of the production periods for these items. Constraints include limits on budget, force structure requirements, retirement of equipment, equipment upgrade, and limits on production facilities. The size and complexity of the Phoenix Model when it was used to consider the procurement of tactical wheeled vehicles for the Army over the following 20- or 30-year period limited its implementation to a large mainframe computer. This sizeable requirement for computing power is due to the need to use integer variables for determining the optimal timing of the production campaigns for the various systems.

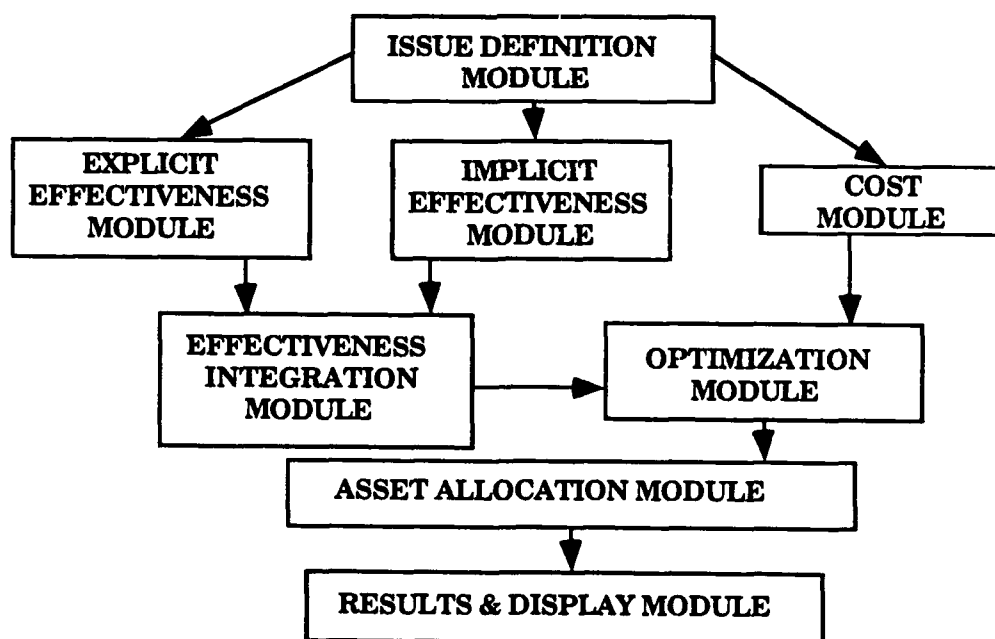
c. A simplification of the Phoenix Model, called Force Modernization Analyzer (FOMOA), was developed at CAA to perform Phoenix-like analyses when the production campaigns for the systems under consideration are given and fixed.<sup>3</sup> FOMOA is a linear program without integer variables, which greatly reduces the computing power required to solve the problem. FOMOA was implemented on a Macintosh IIcx personal computer using spreadsheet software. FOMOA is a relatively fast-running model, and the spreadsheet configuration allows easy data input and output display. FOMOA has been used to produce acquisition strategies for armored systems, wheeled vehicles, and helicopters.

d. Neither Phoenix nor FOMOA considers learning curve costs of the systems to be procured. Schwabauer, et al.<sup>14</sup> suggest the following techniques. In order to address this problem, production quantities are assumed, a priori, likely to be near optimal. Costs are computed that are associated with these quantities which are then used in the budgetary constraints. When the optimization is run, new production quantities are computed which may or may not resemble the a priori quantities. Attempts have been made to iteratively produce cost and quantities in hopes of convergence. However, no such convergence has been shown to be guaranteed.

**CHAPTER 2****VALUE ADDED ANALYSIS METHODOLOGY****2-1. INTRODUCTION**

a. Phoenix and FOMOA have been used to produce acquisition strategies for various systems such as helicopters or trucks. The need arose to provide optimized acquisition strategies across system types. The Value Added Analysis (VAA) methodology was developed by CAA to provide these strategies as well as other analysis to support decisionmaking necessary to build the Army budget. The objective of the VAA optimization module is to maximize the effectiveness of the force subject to constraints on budget, force structure, and production capability.

b. The VAA methodology is modular, and each module performs a specific function. Different tools can be used to perform the function of each module depending upon the analytical requirements established by the issue to be examined. Figure 2-1 shows the various modules and their interrelation. A brief description of the modules follows. The optimization module is described in detail in Chapters 3, 4, and 5. For a more complete description of VAA, see Koury.<sup>12</sup>



**Figure 2-1. VAA Modules**

**2-2. ISSUE DEFINITION.** The purpose of the Issue Definition Module is to refine the problem and its associated elements to be studied so that the data collection and analysis efforts can be focused on the questions and issues of interest to decisionmakers. Issue definition is a process that continues for the duration of the Value Added Analysis. It establishes the general context of the study in terms of the systems and programs to be analyzed, as well as timeframes and scenarios of interest. The module also encompasses the process of clarifying the specific questions asked by the decisionmakers.

**2-3. EXPLICIT AND IMPLICIT EFFECTIVENESS.** Systems effectiveness is measured in two ways. In the Explicit Effectiveness Module, the systems of interest are portrayed in a combat simulation, and their contribution to force level performance is measured. Not all pertinent criteria that bear on the procurement decision are measurable in this manner. The purpose of the Implicit Effectiveness Module is to quantify these hard-to-measure factors. These factors might include political risk, impact on sustainability, and programmatic, as well as other criteria that cannot be directly measured at present. Evaluations are made by individuals who are experts in these criteria. The criteria are assigned weights of relative importance based on a survey of senior Army decisionmakers. Subject matter experts then evaluate (score) how well a system fares in light of these criteria.

**2-4. EFFECTIVENESS INTEGRATION.** When the programs of interest are finally evaluated, a vector of effectiveness scores for the various criteria is obtained for each system. The purpose of the Effectiveness Integration Module is to reduce this vector of information to a single measure. The Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is currently used for this purpose. This method is described in detail by Hwang and Yoon.<sup>7</sup> The single measure obtained for each system can then be used to compare the systems in question with respect to their effectiveness. These measures are used to form the objective function coefficients for the VAA optimization.

**2-5. COST.** Parallel to the determination of the effectiveness of the system in question is the determination of system costs. A complete analysis of all the life cycle costs for each system is performed. For the purpose of developing an acquisition strategy, the portion of the life cycle cost of interest is the funds that would be available for research, development, and acquisition (RDA). In the cost module, data are also collected describing the various components of the RDA costs: fixed costs, variable costs with learning curve effects, and variable costs without learning. These are the costs that are used to build the budgetary constraints for the VAA Optimization Module.

## CHAPTER 3

### OBJECTIVE FUNCTION AND BUDGET CONSTRAINTS IN VALOR

#### 3-1. INTRODUCTION

a. As mentioned, the purpose of VALOR is to produce a "good" and feasible acquisition strategy for the procurement of weapon systems and equipment over a 15-year period. This model utilizes a formulation similar to those of Phoenix and FOMOA and can be represented as follows.

Maximize: Force effectiveness  
 Subject to: Budget ceiling  
               Force structure requirements  
               Production limitations

b. Since the VAA methodology was designed to provide analytical support for the Planning, Programing, Budgeting, and Execution System (PPBES) of the Army Staff, accurate representations of the various categories of constraints are required. Particularly important are the cost values that are used in the budgetary constraints. Thus arose the need for the model to relate cost with quantity automatically as the algorithm is performed. The method developed to incorporate learning curve costing into the optimization formulation is presented in this chapter.

**3-2. OBJECTIVE FUNCTION.** The objective of VALOR, and indeed the Value Added Analysis in general, is to suggest a mix of systems for procurement that will be as effective as possible in combat, subject to constraints on budget, force structure, and production capabilities. The effectiveness of the various candidate systems is evaluated and quantified in the Explicit and Implicit Effectiveness Modules of the VAA methodology, and these various measures are integrated using TOPSIS in the Effectiveness Integration Module. The result is a single measure of a system's contribution to the effectiveness of the overall force for each year the system will be in the force. This measure is then used to form the objective function coefficient,  $v_{ij}$ , which is the per-item contribution of the system to force effectiveness. Let  $x_{ij}$  be defined as the quantity of system  $i$  procured in year  $j$ , where  $j=1, \dots, n$ , with  $n$  being the number of years in the planning horizon. The objective function can then be written as

$$\text{Maximize } \sum_{j=1}^n \sum_{i=1}^m v_{ij} x_{ij}$$

We will consider  $m$  systems in the analysis.

### 3.3. BUDGETARY CONSTRAINTS

**a. Introduction.** We note at the outset that there will be a separate budgetary constraint for each year in the time period of interest for the study. Funds designated for use in a particular year cannot be carried over into following years. The model will maximize effectiveness which is accumulated in the objective by the procurement of equipment through the expenditure of funds. The funds available in each year will be specified in these budgetary constraints, so the accuracy of the various cost components is extremely important to obtaining a valid solution. In this section we develop an approximate method for representing the learning curve costs that will ensure this accuracy.

**b. Notation.** We introduce the following notation.

$\hat{c}_{ij}$  = average cost of a unit of system  $i$  in year  $j$ ,

$Q_{ij}$  = lot midpoint for system  $i$  in year  $j$ ,

$A_i$  = first unit cost of system  $i$ ,

$b_i$  = cost / quantity slope coefficient,

$x_{ij}$  = number of system  $i$  produced in year  $j$ ,

$y_{ij}$  = cumulative number of system  $i$  produced through year  $j$

$$= \sum_{k=1}^j x_{ik},$$

$c_{ij}$  = average cost of a unit of system  $i$  when  $y_{ij}$  items are made in one lot,

$B_j$  = total procurement budget available in year  $j$ ,

$n$  = number of years in planning horizon, and

$m$  = number of systems to be considered.

#### c. Formulation of the Budget Constraint

(1) Using the power form of the learning curve, we have the following expression for the average unit cost for system  $i$  in year  $j$ , when we consider the quantity manufactured in year  $j$  as the "lot" for that year.

$$\hat{c}_{ij} = A_i Q_{ij}^{-b_i}, \text{ where} \quad (1)$$

$$Q_{ij} = \text{lot midpoint} = \frac{F_{ij} + L_{ij} + 2\sqrt{F_{ij}L_{ij}}}{4}, \text{ with}$$



$F_{ij}$  = accumulated number of the first item of type  $i$  produced in year  $j$ , and  $L_{ij}$  = accumulated number of the last item of type  $i$  produced in year  $j$ . We note here that this lot midpoint formula is itself an approximation. However its use is extremely common, and the derivation of the learning curve parameters were based on this formula. Therefore, we constrained ourselves to its use to insure consistency with the given data. Noting that  $L_{ij} = y_{ij-1} + x_{ij}$  and  $F_{ij} = y_{ij-1} + 1$ , we have

$$\hat{c}_{ij} = A_i \left( \frac{y_{ij-1} + 1 + y_{ij-1} + x_{ij} + 2\sqrt{(y_{ij-1} + 1)(y_{ij-1} + x_{ij})}}{4} \right)^{-b_i} \quad (2)$$

Thus, if we consider only the learning curve components of the costs, the budget constraint would take the following form:

$$\sum_{i=1}^m A_i \left( \frac{y_{ij-1} + 1 + y_{ij-1} + x_{ij} + 2\sqrt{(y_{ij-1} + 1)(y_{ij-1} + x_{ij})}}{4} \right)^{-b_i} x_{ij} \leq B_j, \quad j = 1, \dots, n. \quad (3)$$

(2) Ideally, this nonlinear form of the constraint would be used in the optimization. However, the size of the problem makes this approach impractical. Each term of these constraints is a function of several decision variables, making it impossible to approximate the function in a piecewise linear manner. Separability is required for this type of approximation.

(3) In order to overcome the problem of the terms of the constraints not being separable, the approach will be to reformulate the constraint in a way that will make the terms separable, and then correct for any error that arises as the result of the change in form of the constraints.

(4) We would like to find an expression for the cost term that is separable. Consider the following approximation. Let

$$c_{ij} = A_i \left( \frac{1 + y_{ij} + 2\sqrt{y_{ij}}}{4} \right)^{-b_i}, \quad (4)$$

the average unit cost of making  $y_{ij}$  items in one lot. This expression is different from the one in (2) because it ignores the individual years and takes the entire production quantity as one lot. Now, noting that  $x_{ij} = y_{ij} - y_{ij-1}$ , rewrite the budget constraint as

$$\sum_{i=1}^m (c_{ij}y_{ij} - c_{ij-1}y_{ij-1}) \leq B_j, j = 1, \dots, n, \quad (5)$$

where

$$c_{ij-1} = 0,$$

when  $j=1$  or  $j$  represents the first year of production. Notice that (4) is equivalent to (2) for the first lot.

(5) We now examine the difference between the constraints in (3) and those in (5). Our purpose is to make an adjustment to (5) so that it will be equivalent to (3). Then, using  $y_{ij}$  as the decision variables, we will have separable terms in the budget constraints that can be approximated in a piecewise linear manner. For each system,  $i$ , we have

$$\begin{aligned} c_{ij}y_{ij} - c_{ij-1}y_{ij-1} &= c_{ij}(y_{ij-1} + x_{ij}) - c_{ij-1}y_{ij-1} \\ &= c_{ij}x_{ij} - (c_{ij} - c_{ij-1})y_{ij-1}. \end{aligned} \quad (6)$$

Letting

$$c_{ij} = \hat{c}_{ij} + K_{ij}, \text{ where } K_{ij} \text{ is the difference between the two costs,} \quad (7)$$

we then have

$$\begin{aligned} c_{ij}y_{ij} - c_{ij-1}y_{ij-1} &= (\hat{c}_{ij} + K_{ij})x_{ij} - (c_{ij-1} - c_{ij})y_{ij-1} \\ &= \hat{c}_{ij}x_{ij} + K_{ij}x_{ij} - (c_{ij-1} - c_{ij})y_{ij-1}. \end{aligned} \quad (8)$$

Therefore, the original and exact constraint (3) is shown to be equivalent to a corrected version of (5) as follows.

$$\sum_{i=1}^m \hat{c}_{ij}x_{ij} \leq B_j \Leftrightarrow \sum_{i=1}^m \left( c_{ij}y_{ij} - c_{ij-1}y_{ij-1} - (K_{ij}x_{ij} - (c_{ij-1} - c_{ij})y_{ij-1}) \right) \leq B_j \quad (9)$$

So, in order to achieve this equivalence, we have introduced the correction term

$$D_{ij} = K_{ij}x_{ij} - (c_{ij-1} - c_{ij})y_{ij-1}. \quad (10)$$

We note that  $D_{ij} = 0$  for the first year of production because  $K_{ij} = 0$  and  $y_{ij-1} = 0$  in that time period.

#### d. Characterization of the Correction Term

(1) The question remains, are we any better off now than we were before? Although the costs are now functions of only one decision variable, allowing for piecewise linear approximation of the nonlinear function, we still have the error correction terms,  $D_{ij}$ ,  $i = 1, \dots, m$ ,  $j = 1, \dots, n$ , which are even more complicated functions of the quantity variables. We now examine these terms. Recall that

$$D_{ij} = K_{ij}x_{ij} - (c_{ij-1} - c_{ij})y_{ij-1}.$$

Expanding, we have

$$\begin{aligned} D_{ij} = A_i \left( \frac{1 + y_{ij} + 2\sqrt{y_{ij}}}{4} \right)^{-b_i} y_{ij} - A_i \left( \frac{1 + y_{ij-1} + 2\sqrt{y_{ij-1}}}{4} \right)^{-b_i} y_{ij-1} \\ - A_i \left( \frac{y_{ij-1} + 1 + y_{ij} + 2\sqrt{(y_{ij-1} + 1)y_{ij}}}{4} \right)^{-b_i} x_{ij}. \end{aligned} \quad (11)$$

Examining these terms, we have observed the following. First, that these correction terms are very small in magnitude when compared with the cost of producing the corresponding item of equipment for a year, i.e.,

$$\hat{c}_{ij}x_{ij} \gg K_{ij}x_{ij} - (c_{ij-1} - c_{ij})y_{ij-1}.$$

Thus, the terms of (11) almost cancel each other out over the relevant range of the parameters describing the learning curves for systems we have encountered so far. Second, we have noted by examining the learning curves for the systems that are candidates for procurement in the VAA Study that the proportion of the difference between the cost of producing the cumulative number of items as one lot through year  $j$  and that of year  $j-1$  formed by  $D_{ij}$  is almost constant. That is, letting  $M_{ij}$  be this correction proportion, we have

$$M_{ij} = \frac{D_{ij}}{c_{ij}y_{ij} - c_{ij-1}y_{ij-1}} \sim \text{constant}, \quad (12)$$

for given values of  $A_i$  and  $b_i$  for all but the first year of production, over relevant ranges of  $x_{ij}$  for all the systems that we have seen. An exception exists in the first year of production. In that year,  $M_{ij} = 0$  since the approximation is exact. Figures 3-1a and 3-1b give examples of these observations for two systems whose costs were described by learning curves in the Value Added Analysis.

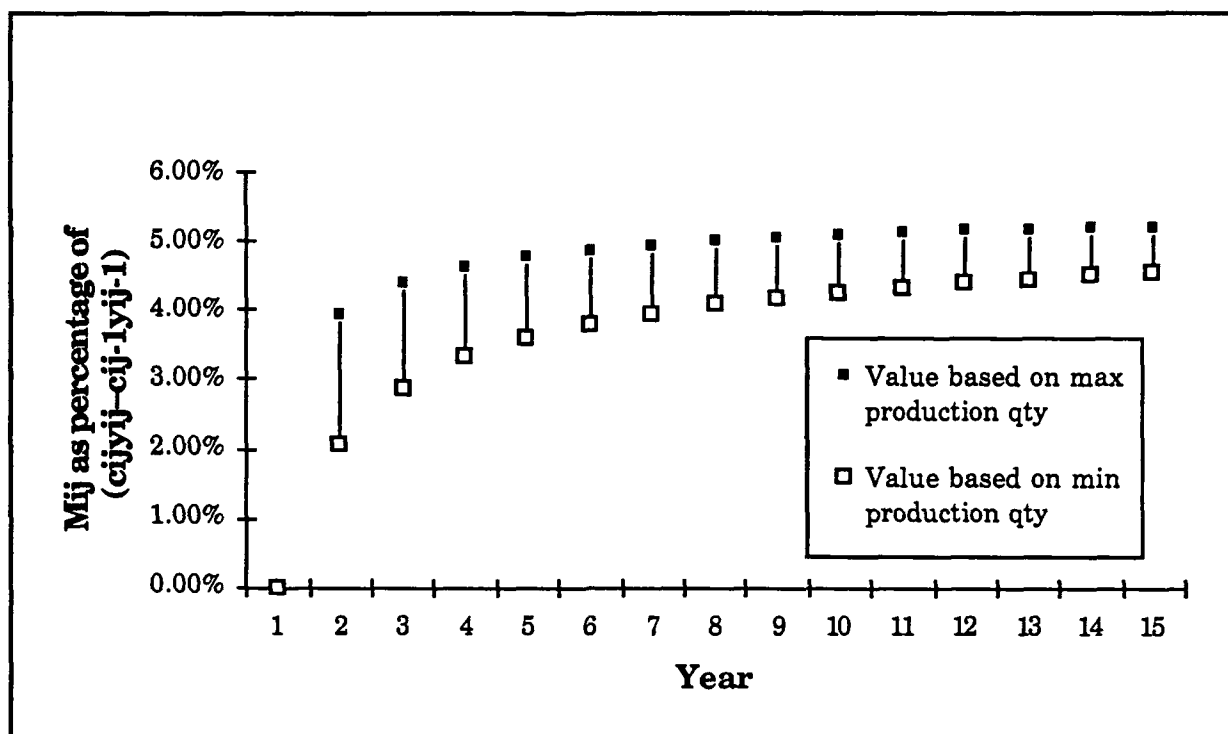


Figure 3-1a. Example of  $M_{ij}$  Values ( $A_i=3.28$ ,  $b_i=.258$ )

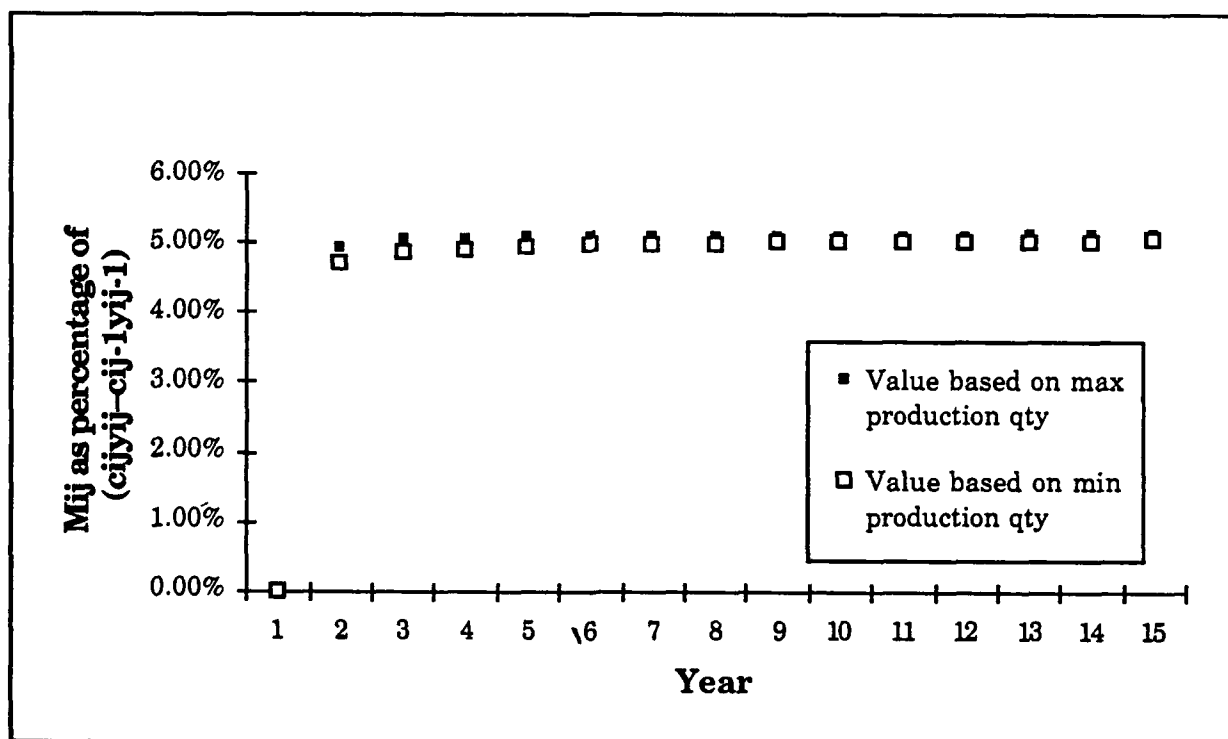


Figure 3-1b. Example of  $M_{ij}$  Values ( $A_i=.1796$ ,  $b_i=.1901$ )

(2) Figure 3-1a shows an example of a learning curve system that has a fairly severe curvature. Note that even in this extreme case, the worst case error is small. Figure 3-1b represents a more typical case where  $M_{ij}$  is almost constant over the years. The consistency of these values is surprising, since the yearly quantities vary from 1,200 to 20,000.

(3) We estimate the value of each correction proportion,  $M_{ij}$ , by computing the upper and lower feasible cumulative production quantities,  $y_{ij}$ , that arise by summing the corresponding maximum feasible values of the production quantities,  $x_{ik}$ ,  $k=1, \dots, j$ , and summing the minimum feasible values of those same quantities. So we have

$$y_{ij} \in [y_{ij\min}, y_{ij\max}] \text{ where}$$

$$y_{ij\min} = \sum_{k=1}^j x_{ik\min} \text{ and } y_{ij\max} = \sum_{k=1}^j x_{ik\max}.$$

Using (10) and (12), and substituting  $y_{ij\min}$  and  $y_{ij-1\min}$  for  $y_{ij}$  and  $y_{ij-1}$ , respectively, we obtain  $M_{ij\min}$ . Following the same procedure, but this time using the maximum values of  $y_{ij}$  and  $y_{ij-1}$ , we get  $M_{ij\max}$ . Empirically, we have found that the arithmetic average of these two quantities gives a good value for  $M_{ij}$ . That is

$$M_{ij} = \frac{M_{ij\max} + M_{ij\min}}{2}. \quad (13)$$

Using this method, we note that the error for each production year by applying this correction would be less than 1 percent for the system shown in Figure 3-1a, and within 2 percent for all the candidate systems examined in the VAA Study.

(4) We can now rewrite the learning curve cost terms (3) using (13) in the following manner. For each year  $j$ , we have:

$$\sum_{i=1}^m \hat{c}_{ij} x_{ij} \leq B_j \Leftrightarrow \sum_{i=1}^m (1 - M_{ij})(c_{ij} y_{ij} - c_{ij-1} y_{ij-1}) \leq B_j. \quad (14)$$

By using this approximation and by using  $y_{ij}$  as the decision variable, these learning curve terms become separable, and although they are still nonlinear, they can be dealt with using a piecewise linear approximation.

(5) We check the computed cost of the program using the cost formula in (2). We have found that the amount by which the approximate cost deviates from the true cost is within 1 or 2 percent for each year. This amount of error is well within that of the available data.

### e. Piecewise Linear Approximation of Learning Curve Term

(1) Now that the learning curve cost terms have been made separable, we can approximate them as a sequence of linear pieces. The technique used here is standard and is described by Bradley, et al.<sup>1</sup> Recall that each cost term is of the form

$$f(y_{ij}) = A_i \left( \frac{1 + y_{ij} + 2\sqrt{y_{ij}}}{4} \right)^{-b_i} y_{ij}. \quad (15)$$

This function is graphed for one of the systems included in the Value Added Analysis, and is shown in Figure 3-2. This system is the same one that was discussed with regard to Figure 3-1(a).

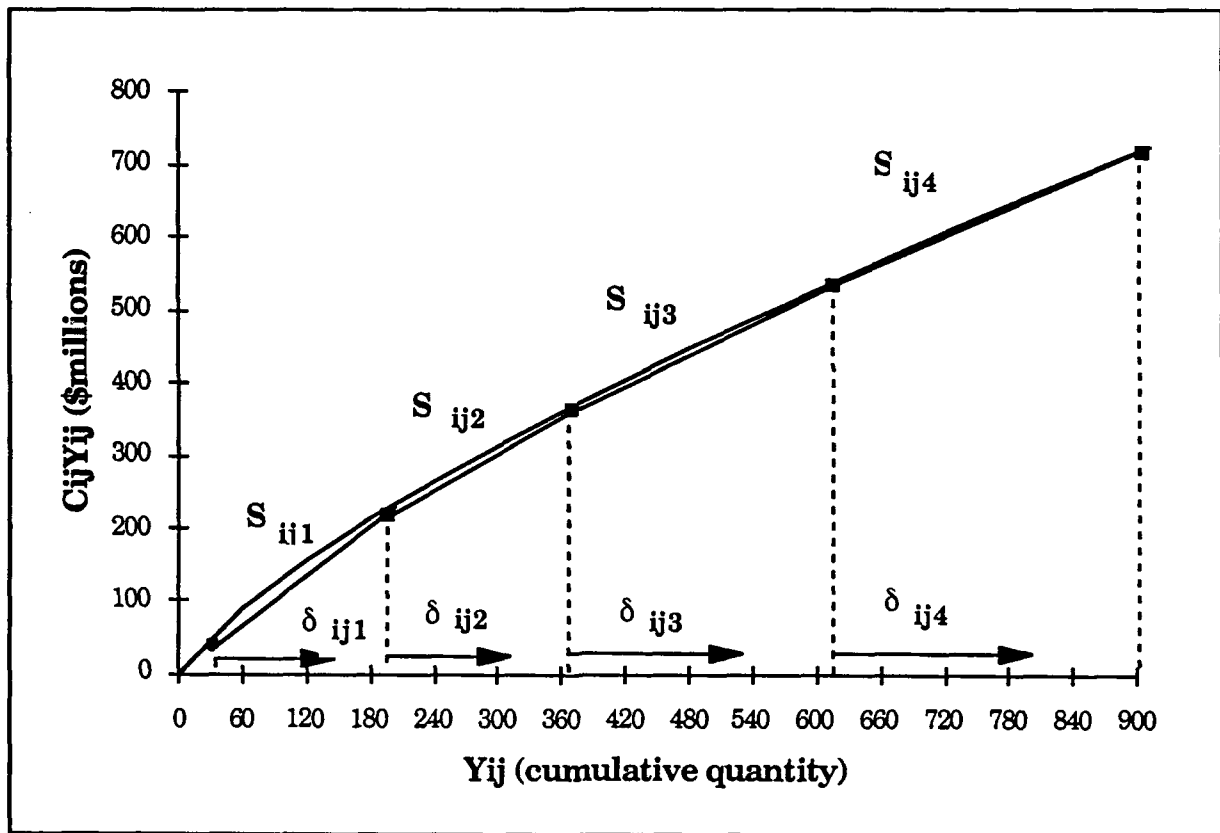


Figure 3-2. Piecewise Approximation of Cost Curve

(2) It is easy to show that this function is concave. It is also very smooth. We have found that over the range of possible values of  $y_{ij}$  that the number of segments needed to approximate this curve varies depending on the parameters of the learning curve and the range of feasible values for  $y_{ij}$ . The relevant range of  $y_{ij}$  is defined as follows. The minimum value of  $y_{ij}$  is the

minimum production quantity in the first year of production. Define this quantity as  $\mu_{ij0}$ . The maximum value of  $y_{ij}$  is the sum of the maximum production quantities over all the years of production. Define this maximum quantity as  $\mu_{ijp}$ , where  $p$  = the number of segments in the approximation.

(3) To find the end points of the line segments that approximate  $f(y_{ij})$ , we find that values of  $y_{ij}$ ,  $\mu_{ij1}$ ,  $\mu_{ij2}$ , ...,  $\mu_{ijp-1}$  such that the following relationship of the derivatives holds.

$$f'(\mu_{ij1}) - f'(\mu_{ij0}) = f'(\mu_{ij2}) - f'(\mu_{ij1}) = \dots = f'(\mu_{ijp}) - f'(\mu_{ijp-1}).$$

(4) Next, we define the following variables.

$\delta_{ijk}$  = the amount greater than  $\mu_{ijk}$ , where

$$0 \leq \delta_{ijk} \leq \mu_{ijk} - \mu_{ijk-1}; k = 1, \dots, p;$$

so that

$$y_{ij} = \mu_{ij0} + \delta_{ij1} + \dots + \delta_{ijp}. \quad (16)$$

$$w_{ijk} = \begin{cases} 1, & \text{if } \delta_{ijk} = \mu_{ijk} - \mu_{ijk-1}, \\ 0, & \text{if } \delta_{ijk} < \mu_{ijk} - \mu_{ijk-1}. \end{cases} \quad (17)$$

(5) The purpose of these binary variables,  $w_{ijk}$ , is to ensure that  $\delta_{ijk}$  will never be positive unless  $\delta_{ijk-1}$  is at its maximum, for  $2 \leq k \leq p$ . These conditions are enforced through the use of the following constraints.

$$\begin{aligned} (\mu_{ij1} - \mu_{ij0})w_{ij1} &\leq \delta_{ij1} \leq \mu_{ij1} - \mu_{ij0}, \\ (\mu_{ij2} - \mu_{ij1})w_{ij2} &\leq \delta_{ij2} \leq (\mu_{ij2} - \mu_{ij1})w_{ij1}, \\ &\cdot \\ &\cdot \\ &\cdot \\ &\cdot \\ (\mu_{ijp-1} - \mu_{ijp-2})w_{ijp-1} &\leq \delta_{ijp-1} \leq (\mu_{ijp-1} - \mu_{ijp-2})w_{ijp-2}, \\ 0 \leq \delta_{ijp} &\leq (\mu_{ijp} - \mu_{ijp-1})w_{ijp-1}. \end{aligned} \quad (18)$$

(6) Notice that the binary variables,  $w_{ijk}$ , act as switches for the  $\delta_{ijk}$  variables such that, when  $\delta_{ij1}$  is pushed to its maximum allowable value,  $\mu_{ij1} - \mu_{ij0}$ ,  $w_{ij1}$  is toggled from a value of 0 to a value of 1. Thus,  $\delta_{ij2}$ , previously constrained to be zero, is allowed to grow toward its own maximum value. Now, calculate the slopes of each segment. Let

$$S_{ijk} = \frac{f(\mu_{ijk}) - f(\mu_{ijk-1})}{\mu_{ijk} - \mu_{ijk-1}}. \quad (19)$$

Then we can approximate each cost term as

$$c_{ij}y_{ij} = f(y_{ij}) - f(\mu_{ij0}) + \sum_{k=1}^p S_{ijk}\delta_{ijk}, \quad (20)$$

with the additional constraints that

$$y_{ij} = \mu_{ij0} + \sum_{k=1}^p \delta_{ijk}, \quad \forall i, j. \quad (21)$$

The approximation in (14) is then written as

$$\sum_{i=1}^m (1 - M_{ij}) \left( f(\mu_{ij0}) + \sum_{k=1}^p S_{ijk}\delta_{ijk} - f(\mu_{ij-10}) - \sum_{k=1}^p S_{ij-1k}\delta_{ij-1k} \right) \leq B_j, \quad (22)$$

with constraints (18) and (21), and using the convention that  $\mu_{i00}$  and  $\delta_{i00}$  are defined to be zero.

(7) Finally, there is no need to explicitly use the  $y_{ij}$  variables. Rather, we impose the following constraints to relate the  $x_{ij}$  variables with the  $\delta_{ij}$  variables. Recalling that

$$\sum_{k=1}^j x_{ij} = y_{ij}, \text{ and using (21), we have}$$

$$\sum_{k=1}^j x_{ij} = \mu_{ij0} + \sum_{k=1}^p \delta_{ijk}. \quad (23)$$

(8) This approximation, then, introduces  $2p-1$  variables, of which  $p-1$  are binary, and  $2p$  constraints to the formulation of the problem for each  $y_{ij}$  variable. The dimensionality of the problem is thus greatly increased.

**f. Systems without Learning Effects.** Not all the systems that are being evaluated exhibit learning behavior. For these systems, an average unit cost,  $\bar{q}_j$ , is specified for each year of production. The cost term associated with these systems have the form  $\bar{q}_j x_{ij}$ , for each year,  $j$ , that system  $i$  is produced. These terms are then included in the budgetary constraints for appropriate years. Some systems have a component of cost that is more appropriately



described as a "nonlearning" variable cost. For these instances, the cost of the system can be described as having both a learning and nonlearning component, and the term  $\tilde{q}_{ij}x_{ij}$  can be introduced together with the learning cost in the budgetary constraint.

**g. Consideration of Fixed Costs.** Some costs, such as research, development, test and evaluation (RDTE),  $R_{ij}$ , expenditures, or nonrecurring fixed manufacturing costs,  $V_{ij}$ , are not incurred on a per-unit basis. These costs simply reduce the funds available for procurement (lower the values of the  $B_j$ ) over the years they are expended. However, when the model is used to evaluate cuts or cancellation of programs, significant savings can be accrued by recouping the RDTE or other fixed cost funds that have not yet been spent. Thus, these costs must be tied to the programs being evaluated in a meaningful way. The discussion of how to handle the evaluation of potential program cancellations will appear later in this paper.

#### **h. Production Campaigns**

(1) Typically, each system is not procured in every year of the planning horizon. We want to be able to handle the situation in which the various systems are produced in only a subset of the years in the planning horizon. We assume that the years of production for each system, the so-called production campaign, is given and fixed. This assumption represents a simplification of the formulation used in the Phoenix Model which allows the optimization to pick the best production campaign for the systems. Extending the VAA optimization formulation to choose an optimal production campaign for each system will be left as future work.

(2) In order to make this modification, we introduce the following notation. Let  $t_i$  = first year of production of system  $i$ , and let  $n_i$  = number of years in production campaign for system  $i$ . Then define set  $I_j$  such that

$$I_j = \{i \in \{1, \dots, m\} : \text{system } i \text{ is produced in year } j\}.$$

Including fixed and nonlearning costs, the constraints (22) and (23) become

$$\begin{aligned} \sum_{i \in I_j} \left[ (1 - M_{ij}) \left( f(\mu_{ij0}) + \sum_{k=1}^p S_{ijk} \delta_{ijk} - f(\mu_{ij-10}) - \sum_{k=1}^p S_{ij-1k} \delta_{ij-1k} \right) + \tilde{c}_{ij} x_{ij} \right] \\ + \sum_{i=1}^m (R_{ij} + V_{ij}) \leq B_j, \quad \forall j = 1, \dots, n, \\ \sum_{k=t_i}^j x_{ik} = \mu_{ij0} + \sum_{k=1}^p \delta_{ijk}, \quad \text{for all } t_i \leq j \leq t_i + n_i - 1, \text{ for all } i = 1, \dots, m. \end{aligned} \quad (24)$$

with constraints (18), and using the convention that  $\mu_{i00}$  and  $\delta_{i00}$  are defined to be zero. One of these constraints will appear for each year in the planning horizon. We note here that  $R_{ij}$  and  $V_{ij}$  will be zero in years that no such costs exist.

## CHAPTER 4

## OTHER FEATURES OF THE VAA OPTIMIZATION MODEL

**4-1. INTRODUCTION.** The quantities of items to be procured must be consistent with requirements for these items in the various Army units as specified in the given force structure. Constraints are also imposed on the number of items that can be produced during each time period, which reflect the capacity of the production facilities. Both force structure and production constraints are explained below. In addition, the optimization must be used to evaluate program cuts and fund reallocation. The scheme used to facilitate these analyses is also described in this chapter.

**4-2. FORCE STRUCTURE REQUIREMENTS**

**a. Introduction.** Force structure requirements drive the decisions of how many of a particular item of equipment should be procured, and on the identification of the particular Army units that will receive the equipment when it is fielded. In order to make these decisions, a set of four force packages is identified. Force Packages are prioritized groupings of units that specify the order in which newly procured equipment is fielded. The units contained in Force Package I would be fielded first, followed by those in Force Package II, III, and IV, as long as sufficient funds are available. The study sponsor must specify the level of force structure to be considered. For example, the sponsor might know that insufficient funds are available to buy enough for all the force packages, so he may designate that Force Package I must be filled with equipment for all systems, and that nothing would be procured beyond those needed to fill Force Package II. Thus, the force structure bounds for each system would be established. It would then be known what the minimum and maximum allowable procurements are for each system by the end of each system's production campaign.

**b. Equal Quantity Representation.** There are two ways to represent these constraints in the optimization model. The first involves constraining each year's procurement quantity of each system to be the following:

$$\frac{F_{i\min}}{n_i} \leq x_{ij} \leq \frac{F_{i\max}}{n_i}, \forall \text{ systems } i \text{ produced in year } j, \forall j,$$

where, for system  $i$ ,

$F_{i\min}$  = minimum force structure requirement,  
 $F_{i\max}$  = maximum force structure requirement,  
 and  $n_i$  = number of years of production.

The advantage of this method is that the series of production quantities over the production campaign tends to be more stable. From a practical standpoint, avoiding wide swings in annual production quantities is desirable. The disadvantage is that the constraint is tighter, limiting the flexibility of the model to find a better solution.

**c. Total Quantity Representation.** The second option involves constraining the sum of all items produced over the entire production campaign to be between the force structure minimum and maximum. That is,

$$F_{i_{\min}} \leq \sum_{j=t_i}^{t_i+n_i-1} x_{ij} \leq F_{i_{\max}}, \forall i, j.$$

where  $t_i$  = the first year of production for system  $i$ . Constraints on the individual  $x_{ij}$ 's would be based on production capacities. These constraints will be discussed later. This option allows for better solutions with respect to the effectiveness objective but tends to give wide swings in the annual production quantities. The decision regarding which of these schemes to use rests with the study sponsor. We note that the choice of schemes can be made on a system-by-system basis, maximizing the flexibility of the model. A modification of this representation is the introduction of additional constraints on the  $x_{ij}$ 's that would be used to force a more stable stream of production quantities. Letting  $\rho_i$  = the allowable variation in yearly production quantities, and noting that we would not apply these constraints during ramp-up years, these constraints would take the form

$$(1 - \rho_i)x_{ij-1} \leq x_{ij} \leq (1 + \rho_i)x_{ij-1}, \forall i, \forall t_i + 1 \leq j \leq t_i + n_i - 1$$

In this case, the production quantity would be constrained to be within  $100\rho_i$  percent of the previous year's quantity, avoiding undesirable swings in production. Of course the selection of the percentage is arbitrary and can be adjusted appropriately on a system-by-system basis.

### 4-3. PRODUCTION CONSTRAINTS

**a. Introduction.** The capacities of the various production facilities must be considered in constraining the production quantities. Typically, the model is constrained to produce at least the minimum sustaining rate (MSR) of production and no more than the maximum production rate (MPR). The minimum sustaining rate is defined as the production rate needed to keep the production line open while maintaining a responsive vendor and supplier base. The MSR is frequently equated to maintaining a warm production base. The MPR is the production rate which maximizes the production capacity of existing tooling or facilities without requiring additional investment to increase the capacity.

**b. Ramp-ups in Production.** Most systems that are not yet in production at the beginning of the planning horizon will have a ramp-up in production over 2 or 3 years where the production capacity is lower than the MSR. These ramp-up years must be considered when constraints on the production quantities are constructed. The ramp-up quantities would be reflected in the bounds on the production quantities in the first few years of the production campaign specified for the particular system in question.

**c. Production and Force Structure.** The production constraints must be used in conjunction with the force structure constraints to develop the upper and lower bounds of the production quantities,  $x_{ij}$ . When using the equal quantity representation of the force structure bounds, some preprocessing is required to ensure that these bounds are applied appropriately. Let

- $P_{minij}$  = lower production bound (MSR) for system  $i$  in year  $j$ ,
- $P_{maxij}$  = upper production bound (MPR) for system  $i$  in year  $j$ ,
- $F_{minij}$  = lower force structure bound for system  $i$  in year  $j$ ,
- $F_{maxij}$  = upper force structure bound for system  $i$  in year  $j$ ,
- $U_{ij}$  = upper bound on  $x_{ij}$  for system  $i$  in year  $j$ , and
- $L_{ij}$  = lower bound on  $x_{ij}$  for system  $i$  in year  $j$ .

The upper and lower bounds on  $x_{ij}$  are determined using the following rules.

- (1) If  $P_{minij} \leq F_{minij}$ , then  $L_{ij} = P_{minij}$ .
- (2) If  $F_{minij} \leq P_{minij}$ , then  $L_{ij} = F_{minij}$ .
- (3) If  $P_{maxij} \leq F_{maxij}$ , then  $U_{ij} = F_{maxij}$ .
- (4) If  $F_{maxij} \leq P_{maxij}$ , then  $U_{ij} = P_{maxij}$ .
- (5) If  $F_{maxij} \leq P_{minij}$ , then  $L_{ij} = U_{ij} = P_{minij}$ .
- (6) If  $F_{minij} \leq P_{maxij}$ , then  $L_{ij} = U_{ij} = P_{maxij}$ .

Note that rules (5) and (6) address the problem of mismatch between the production limitations and the force structure requirements. When these conditions arise, it is prudent to reconsider the length of production campaign. Note also that the above rules only apply to the first alternative for determining force structure constraints. If the second alternative is employed, namely the total quantity representation, the production bounds are used to bound  $x_{ij}$ 's. Care must be taken, however, to ensure that mismatches in the production limitations and force structure requirements are resolved. Otherwise, an infeasible program will result.

#### 4.4. EVALUATION OF PROGRAM CUTS

**a. Introduction.** As mentioned previously, we want the model to be useful in analyzing program cut alternatives which arise during the building of the Army program and budget. In order to do so, two factors must be considered.

(1) First, in order to determine if a system should be procured or not, we must allow a disjoint feasible set. That is, systems that are candidates for exclusion from the program (cuts) must be allowed to be procured in quantities satisfying the above constraints or not to be procured. So both  $x_{ij} = 0$  and  $L_{ij} \leq x_{ij} \leq U_{ij}$  are allowed.

(2) Next, if some fixed costs, such as RDTE costs, are associated with the procurement of the system, whose value is independent of the quantity purchased, then they would also be saved if the procurement of the system is canceled. Previously these costs were accounted for by simply subtracting them from the budget in the appropriate years. Some mechanism was needed to include them, or not, as appropriate.

**b. Implementation.** In order to implement this enhancement to the model, we introduce binary variables defined as follows.

$$u_i = \begin{cases} 1, & \text{if system } i \text{ is procured,} \\ 0, & \text{otherwise.} \end{cases}$$

We then modify the constraints on the  $x_{ij}$ 's as follows.

$$L_{ij}u_i \leq x_{ij} \leq U_{ij}u_i.$$

We note that when system  $i$  is procured, the  $u_i = 1$ , and these constraints become equivalent to those previously discussed. When  $u_i = 0$ , then the value of  $x_{ij}$  is constrained to be zero also.

**c. Fixed Costs.** This binary variable can also be used to switch on and off the fixed costs in the budget constraints. We modify the budget constraint for year  $j$  to be the following.

$$\sum_{i \in J} \left[ (1 - M_{ij}) \left( f(\mu_{ij0})u_i + \sum_{k=1}^p S_{ijk} \delta_{ijk} - f(\mu_{ij-10})u_i - \sum_{k=1}^p S_{ij-1k} \delta_{ij-1k} \right) + \tilde{c}_{ij}x_{ij} \right] + \sum_{i=1}^m (R_{ij}u_i + V_{ij}u_i) \leq B_j.$$

We note that if  $u_i = 1$ , then the available funds for year  $j$  are effectively reduced by the amount of fixed cost expended in that year. If  $u_i = 0$ , then no such

deduction is made. Thus, the RDTE and fixed costs are only included if the system is procured and are ignored otherwise.

**d. Combined System Procurements.** Another important use of these binary  $u_i$  variables is to constrain the model to procure systems in combinations. For example, suppose we are considering the procurement of a new artillery system and a resupply vehicle that will carry its ammunition. We may want to constrain the model to refrain from procuring the resupply vehicle unless it procures the artillery system as well. By introducing the following constraint, we can force the model to relate these systems as described above.

$$u_{\text{resupply}} - u_{\text{artillery}} \leq 0.$$

This constraint ensures that the binary variable associated with the artillery system is greater than or equal to that of the resupply vehicle. Thus, the unreasonable result of procuring the resupply vehicle without the artillery system is avoided. The introduction of this type of constraint in various ways allows the analyst to evaluate different combinations of systems.

**e. Multiple Production Campaigns.** We have previously assumed that the production campaign for each system was fixed. However, we can use the binary  $u_i$  variables to evaluate multiple production campaigns. We do this by introducing the same system with several production campaigns. Then, to ensure that the same system is not included in the program more than once, we use the following constraint,

$$u_1 + u_2 + \dots + u_k \leq 1,$$

where  $k$  = the number of candidate production campaigns for the system in question.

## CHAPTER 5

### IMPLEMENTATION AND PERFORMANCE

#### 5-1. IMPLEMENTATION

**a. Hardware and Software.** The VALOR Model was implemented using the IBM Optimization Software Library (OSL),<sup>10</sup> on an IBM RISC 6000 work station. OSL, as the name suggests, is a set of subroutines that can be called to manipulate and solve a variety of optimization problems. It can be accessed through FORTRAN or C programming languages. The software is flexible with respect to input and accepts either standard MPS input format or allows direct access to the data structure to set up the problem.

**b. Programming.** We elected to write a front-end application program in FORTRAN that reads the data, processes the data into the appropriate data structures for the optimizer, calls the optimization subroutines, and then prints the results.

#### 5-2. PERFORMANCE

**a. Early Tests.** The first test problems with realistic data were of the order of 700 rows with 800 variables, of which 350 were binary. This program evaluated about 20 systems and took about 20 minutes of CPU time to run. The model was also run using the system cutting feature; the running time increased 25 - 50 percent. In these early tests, we limited ourselves to evaluating only a small subset of the systems as candidates for elimination from the Army budget, usually three or four systems. We found that attempting to evaluate too many such systems greatly increased the run time. Although the run time was increased for most of the runs in which this feature was employed, there were instances where increase in run time was minimal.

**b. Preprocessing.** OSL has the facility to preprocess a mixed integer program to reduce the branch and bound tree. We tried preprocessing our program, but no apparent economies were found. In fact, the procedure increased the run time of the model. This increase could be expected, since the preprocessing procedure itself has a processing time on the order of  $k^2$  to  $k^3$ , where  $k$  is the number of nonzero elements in the matrix. Since no economies were found, this preprocessing was wasted time. Nevertheless, since the processing time of the branch and bound algorithm itself is of the order of  $2^n$ , where  $n$  is the number of integer variables, the preprocessor was worth a try.

**c. Performance Tuning.** Although the internal OSL preprocessor failed to find any adjustments to the formulation that would improve its performance, a close examination of the structure of the formulation yielded several performance enhancing adjustments. It is known that to improve the performance of a mixed integer program, efforts that reduce the separation between the solution of the linear programming relaxation of the problem and



the integer solution will improve performance. Also, the power of a branch and bound algorithm can be enhanced if relationships between the integer variables are exploited. In order to accomplish these objectives, additional constraints are introduced. A good discussion of the techniques used to identify the additional constraints is given by Johnson and Nemhauser.<sup>9</sup>

**d. Strong Inequalities.** The first set of constraints introduced was based on the relationship between the binary variables that are used to implement the piecewise linear approximation of cost curve. Recall that we include a cost curve for each system  $i$  for each year  $j$  of production, introducing several binary variables,  $w_{ijk}$ , into the formulation, the number depending on the number of linear pieces employed in the approximation. Since the cost curves are based on the cumulative quantity of system  $i$  procured over the years,  $y_{ij}$ , we know that  $y_{ij} \geq y_{ij-1}$ . Thus, since

$$y_{ij} = \mu_{ij0} + \sum_{k=1}^p \delta_{ijk}, \quad \forall i \in I_j, j = 1, \dots, n,$$

we know that  $\delta_{ijk} \geq \delta_{ij-1k}$  for all  $j > t_i$ . Therefore, we can impose the following constraints on the  $w_{ijk}$  binary variables.

$$w_{ijk} \geq w_{ij-1k}; \text{ for all } j > t_i, i = 1, \dots, m, k = 1, \dots, p.$$

This relation between the  $w_{ijk}$  variables is implicit in the formulation as it has been presented. However, if these constraints are not explicitly imposed, the branch and bound algorithm will branch on each binary variable separately. When these constraints are imposed, branches become much more powerful in the sense that setting one variable to the value 0 potentially sets many others as well. Similarly, we see immediately that all the  $w_{ijk}$  variables will be 0 unless the  $u_i$  variables are set at 1. So, another set of constraints that is implicit in the formulation but whose explicit inclusion improves the performance of the algorithm are expressed as follows:

$$u_i \geq w_{ijk}, \text{ for all } j > t_i, \text{ for all } i = 1, \dots, m, k = 1, \dots, p.$$

These constraints ensure that all  $w_{ijk}$  variables are set to 0 if  $u_i$  is 0, making branches on the  $u_i$  variables very powerful. Approximately 500 additional constraints were added to the formulation.

**e. Improved Performance.** As the result of the additional constraints, the performance of the model was much improved. Final test runs as well as the production runs were significantly larger than the initial tests that were performed. Ultimately, 45 systems were analyzed, of which 22 had learning curve costs. The mixed integer program had about 4,000 rows with 3,000 variables, of which about 500 were binary integers and 5,500 nonzero elements. The run time for this improved formulation was reduced to between 2 and 13 minutes of CPU time on the IBM RISC 6000 320H. Another important aspect of this improved formulation was that we were no longer limited to evaluating a small subset of systems as candidates for elimination. We were able to make the procurement of all the systems optional. This capability became very important in the conduct of the Value Added Analysis, since the main emphasis was on identifying funding tradeoffs among the candidate systems. Without the performance-enhancing modifications, the model would not have been as responsive as was necessary to provide the required analytical support.

## CHAPTER 6

### CONCLUSIONS

**6-1. METHODOLOGY AND COMPUTER RESOURCES.** The methodology introduced in this paper seems to do a good job of incorporating the learning curve effects on costing into the budget constraints of the Value Added Analysis acquisition strategy optimization. The introduction of this feature greatly increases the computational overhead associated with solving problems of this nature. As a result, implementation of this enhancement to acquisition strategy models requires significantly increased computing resources to obtain a solution.

**6-2. APPROXIMATION.** This methodology is an approximation, and checks are necessary to ensure the approximation is accurate enough. In our experience, the approximation has yielded results in which the expended program dollars, calculated using the nonlinear cost function and the optimized quantities, were within 2 percent of the nominal value. Considering the approximate nature of costing systems that will only be procured in the far distant future, 2 percent is adequate. In applications that require more accuracy, the approximation can be made more nearly exact by increasing the number of pieces in the piecewise approximation.

**6-3. APPLICATION.** The use of this methodology has been shown to improve the quality of the optimization for the purpose of acquisition strategy. In this era of tightly constrained budgets for procurement, accurate costing is essential to get the most from limited funds. This methodology has enhanced analytical efforts that help accomplish this task. This optimization model was successfully used to assist the Army Staff in evaluating the various alternative weapon systems considered for procurement. The model was particularly useful in identifying the years in which budget constraints were extremely tight with respect to planned production campaigns, suggesting modifications that could be made to proposed programs. The model was also extremely useful in identifying systems that were excluded from the solution when other systems, or combinations of systems, were forced to be procured. This capability gave the leadership a window into the cost of their decisions as they related to system tradeoffs.

**6-4. CONCLUSION.** VALOR has provided a new dimension to the PPBES process for the Department of the Army Staff. The staff now has available in a single model the capability to pull together data, policy, and guidance quickly and accurately in order to develop a balanced Army program.

## APPENDIX A

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**APPENDIX B**  
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## **VALUE ADDED LINEAR OPTIMIZATION OF RESOURCES (VALOR)**

**SUMMARY  
CAA-TP-92-1**

**THE REASON FOR PERFORMING THIS RESEARCH** was to formulate a math programming algorithm that could be used to perform acquisition strategy optimization that dynamically incorporates nonlinear "learning curve" costs for use in Value Added Analysis. An approximate mixed integer programming (MIP) formulation was devised for this purpose.

**THE SPONSOR** was the Director, US Army Concepts Analysis Agency.

**THE OBJECTIVES** were to:

- (1) Identify the need for dynamic learning curve costing in acquisition strategy optimization.
- (2) Formulate a specific MIP for computer solution.
- (3) Implement performance-improving measures to speed the solution of the model.
- (4) Report on the performance and characteristics of the model implementation.

**THE SCOPE OF THE PAPER** was limited to analysis of the major item systems under consideration for procurement in the Army Program Value Added Analysis 94-99 (VAA Phase II) Study.

**THE MAIN ASSUMPTION** of this work is: learning curve costs can be described as an exponential function of the cumulative number of items produced.

**THE BASIC APPROACH** used in this analysis was to formulate a MIP with the objective of maximizing the effectiveness of the force subject to constraints on budget, force structure, and production capabilities. Additional constraints were added to improve computational performance.

**THE PRINCIPAL FINDINGS** of the work reported herein are:

- (1) Approximate nonlinear learning curve costs can be calculated in a mixed integer programming algorithm.
- (2) The performance of the mixed integer programming model used for cross mission area acquisition strategy is such that extremely fast response can be given to "what-if" type questions from study sponsors.

**THIS EFFORT** was directed by LTC Andrew G. Loerch, Force Systems Directorate.

**COMMENTS AND QUESTIONS** may be sent to the Director, US Army Concepts Analysis Agency, ATTN: CSCA-FSR, 8120 Woodmont Avenue, Bethesda, MD 20814-2797.





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